Weak Homology of Bright Elliptical Galaxies

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Abstract. Studies of the Fundamental Plane of early-type galaxies, from small to intermediate redshifts, are often carried out under the guiding principle that the Fundamental Plane reflects the existence of an underlying mass-luminosity relation for such galaxies, in a scenario where elliptical galaxies are homologous systems in dynamical equilibrium. Here I will re-examine the issue of whether empirical evidence supports the view that significant systematic deviations from strict homology occur in the structure and dynamics of bright elliptical galaxies. In addition, I will discuss possible mechanisms of dynamical evolution for these systems, in the light of some classical thermodynamical arguments and of recent N-body simulations for stellar systems under the influence of weak collisionality.

1 Introduction

This article focuses on three main questions: (1) Are elliptical galaxies structurally similar to each other? (2) Which detailed dynamical mechanisms can make elliptical galaxies evolve? (3) Are there general trends to be anticipated for the evolution of these stellar systems?

Here I will report on a long-term research project aimed at providing answers to the above questions. Some interesting clues have been discovered only very recently [5], [6], [12]. Most of the paper refers to the class of bright ellipticals only; low-luminosity ellipticals are known to be characterized by different dynamical properties.

2 Structure of bright elliptical galaxies

The answer to whether elliptical galaxies can be considered to be structurally similar to each other depends on the specific context in which the question is posed and addressed. Below, I will focus on the context of the physical interpretation of the Fundamental Plane ([26], [23]).

As demonstrated by a number of investigations (e.g., see [29], [30] for a study based on a sample of more than 200 early-type galaxies), the observed correlation that defines the Fundamental Plane, $\log R_e = \alpha \log \sigma_0 + \beta SB_e + \gamma$ (with $\alpha = 1.25 \pm 0.1$, $\beta = 0.32 \pm 0.03$, $\gamma = -8.895$ in the B band; the effective radius being measured in kpc, the central velocity dispersion in km/sec, the mean surface brightness in $mag/arcsec^2$ [2], [29]), is remarkably tight, with a scatter on the order of 15% in R_e.

The following simple physical argument has been put forward as an interpretation of this important physical scaling law. If we note that (1) the observed luminosity law of bright elliptical galaxies appears to be universal (the so-called $R^{1/4}$ law; [22]) and (2) the kinematical structure of these systems is regular and uniform ([4], [28]), it is natural to conclude that elliptical galaxies should be considered as homologous dynamical systems, in the sense that the relevant virial coefficient K_V should be taken to be approximately constant from galaxy to galaxy. Then, (3) given the existence of the virial constraint, the Fundamental Plane can be seen as the manifestation of a mass–luminosity relation for galaxies (see [27], [46]). In fact, the virial theorem can be written as $GM_\star/R_e = L(G/R_e)(M_\star/L) = K_V\sigma_0^2$, where $M_\star$ is the mass of the luminous component and L is the total luminosity. By eliminating σ_0 from the Fundamental Plane relation, one finds:

$$\left(\frac{M_\star}{L}\right)\frac{1}{K_V} \propto R_e^{(2-10\beta+\alpha)/\alpha} L^{(5\beta-\alpha)/\alpha} \sim L^{(5\beta-\alpha)/\alpha}. \quad (1)$$

The latter relation follows from the *empirical* fact that $2 - 10\beta + \alpha \approx 0$.

Unfortunately, there are empirical and theoretical findings that work against the hypotheses at the basis of the previous argument. First of all, significant deviations from the $R^{1/4}$ law have long been noted (see [17], [40]), and found to correlate systematically with the galaxy luminosity (see also [25]). Second, studies that have measured the amount and distribution of dark matter in el-

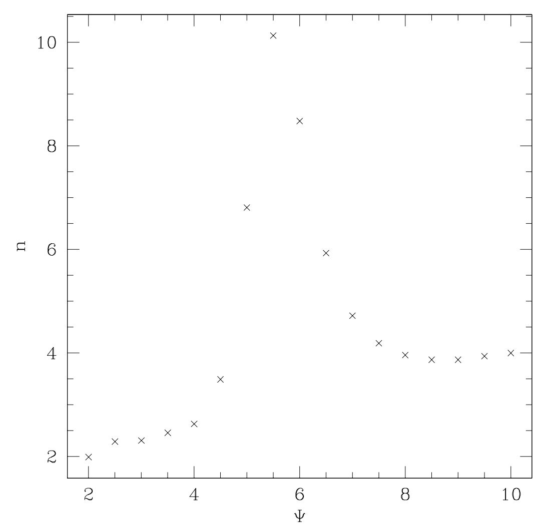

Fig. 1. The best-fit $n(\Psi)$, obtained by fitting the f_∞ models, projected along the line of sight, with $R^{1/n}$ profiles. Note the plateau at $n = 4$ reached by concentrated (high-Ψ) models, for which the radial range adopted in the fit is $0.1 \le R/R_e \le 10$ (from [5])

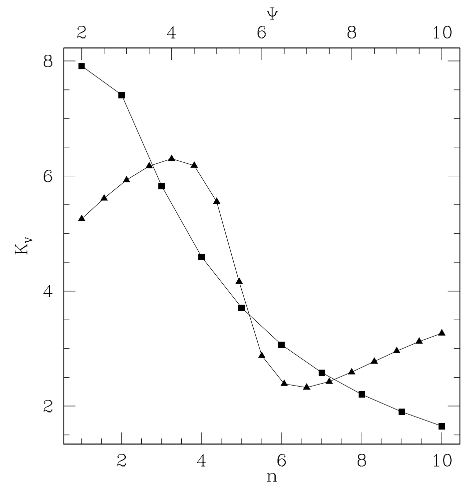

Fig. 2. The virial coefficient for the f_∞ (triangles) and for the isotropic $R^{1/n}$ (squares) models, based on an aperture of radius $R_e/8$ (from [5])

lipticals (see [4]) have shown that the presence of dark matter is more prominent in brighter and spatially larger galaxies, thus demonstrating that the virial coefficient may vary significantly from galaxy to galaxy. A curious theoretical point adds further caution to the perception that ellipticals should be considered homologous systems. This derives from direct inspection of the so-called f_∞ sequence of models [9]. As demonstrated in [5], models that appear to be all (see Fig. 1, for Ψ in the range $7-10$) very well fitted by the $R^{1/4}$ law, over a luminosity range of more than ten magnitudes, may be characterized by significantly different values of the relevant virial coefficient (see Fig. 2, the triangles representing the virial coefficient for the f_∞ sequence of models), as a result of the impact of a more and more concentrated nucleus.

In [5] we have further confirmed, by close inspection of four cases (NGC 1379, NGC 4458, NGC 4374, NGC 4552; studied in great detail by comparing the performance of a number of fitting procedures on data taken from [19], [18]), that the Sersic [41] index n for the $R^{1/n}$ photometric profiles can indeed be very different from 4 (in particular, for NGC 4552 we find $n \approx 11$, with residuals on the order of 0.2 magnitudes; a fit performed with the $R^{1/4}$ law would lead to residuals up to one magnitude, while a fit based on an $R^{1/4}+$ exponential profile would have residuals up to half a magnitude). On the other hand, we have checked that, if the luminosity range where the fit to the photometric profile is performed is reduced to less than 5 magnitudes, then (see [16]) the profiles are indeed well fitted by a “universal” $R^{1/4}$ law.

In conclusion, while we find it necessary to dismiss strict homology as a viable description of elliptical galaxies in relation to the interpretation of the Fundamental Plane, the existence of the empirical scaling law suggests that some kind of *weak homology* must be enforced (expressed by Eq. (1)), as a correlation between structural properties and total luminosity. In [5] we have also proved that a large scatter in the dynamical correlations (e.g., in the $n \sim -19 + 3 \log L$ relation noted in [17], [25]) may well be compatible with the observed tightness of the Fundamental Plane.

3 Mechanisms of dynamical evolution

Given the conclusion that elliptical galaxies have to be considered only weakly homologous systems, it is natural to ask whether and how individual galaxies may change their internal structure via dynamical processes. This general issue is especially important, if we recall that typically, in the study of the cosmological evolution of the Fundamental Plane (see [44] and references therein), strict homology and thus a mass–luminosity relation is assumed for the observed galaxies and an interpretation of the data (see Fig. 3) is made in terms of pure *passive evolution* (through the evolution of the luminosity resulting from the evolution of the properties of stellar populations).

Besides the possibility of major merger events, are there significant sources of dynamical evolution for elliptical galaxies to be considered? As noted recently [3], the traditional approach to the study of elliptical galaxies, in terms of equilibrium and stability for the solutions of the collisionless Boltzmann equation, supplemented by the Poisson equation, may be misinterpreted. Given the very large values of typical star-star relaxation times in elliptical galaxies (see [20], [42]) it is generally taken for granted that, unless a system happens to be in a dynamically unstable state (for example, a condition of excessive radial anisotropy; see [39]), its state is basically "frozen" into an equilibrium distribution function. Thus the only task left to the dynamicist would be to decipher which distribution function best describes the observed states (a task that is particularly difficult for non-spherical systems) taken to be strictly stationary.

In our opinion, the above picture is oversimplified and may lead to an improper perception of the dynamics of real stellar systems. If, for simplicity, we take the view that elliptical galaxies have formed via collisionless collapse (see [45]), we should realize that splitting past and present conditions (that is formation processes and a collisionless equilibrium state) is just an idealization that the theory makes in order to define a basic state and to study its properties. In reality, stellar systems evolve continually and we should check to what extent the evolution processes change the internal structure of galaxies.

There are several specific mechanisms and causes for dynamical evolution that could be studied: (i) "Granularity" in phase space left over from the initial collapse; (ii) Presence of gas in various phases, especially of the hot X-ray emitting interstellar medium; (iii) Interactions with a compact central object;

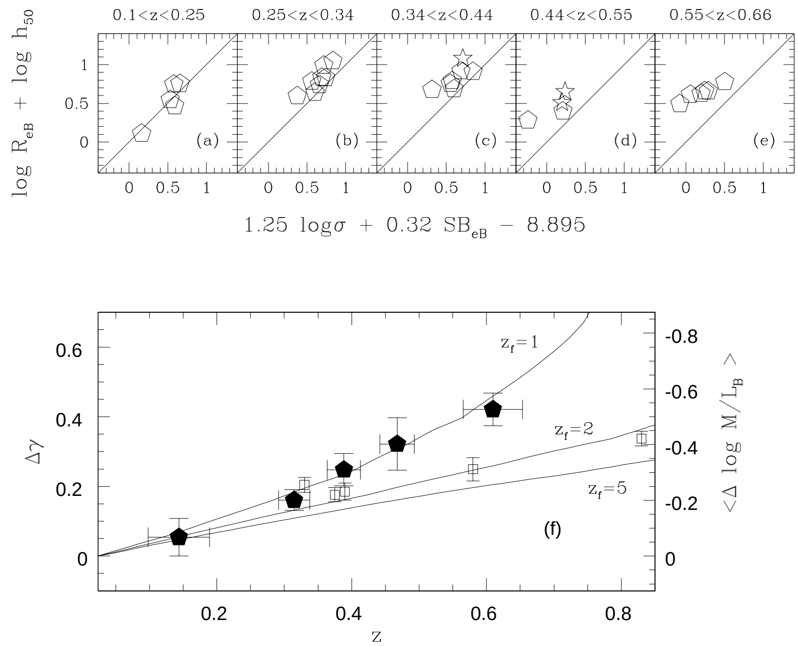

Fig. 3. The Fundamental Plane in the rest frame B band. In panels (a) to (e), field E/S0 galaxies are shown, binned in redshift, and compared to the Fundamental Plane found in the Coma Cluster by [2]. Panel (f) shows the average offset of the intercept of field galaxies from the local Fundamental Plane relation as a function of redshift (large filled pentagons) compared to the offset observed in clusters (open squares). The solid lines represent the evolution predicted for passively evolving stellar populations formed in a single burst at $z = 1, 2, 5$ (from top to bottom). This figure is taken from [44] where full references are given to the sources for cluster data points and stellar synthesis models

(iv) Interactions between the galaxy and its own globular cluster system; (v) Interactions with external satellites and effects of tides and minor mergers.

In a recent paper [6] we have tried to quantify the role of items (iv) and (v) above by means of N-body simulations. The idea at the basis of these studies is that heavy objects can suffer dynamical friction and then be dragged in toward the galaxy center, as studied earlier, for example, in [15], [14], [48]; in fact, the parallel momentum transport relaxation time T_{fr} is related to the deflection relaxation time T_D by a factor that can be very small when a heavy test particle moves through a field of lighter particles: $T_{fr} = 2T_D m_f/(m_t + m_f)$. We have thus revisited the problem of simulating the orbital decay of a satellite, placed initially on a circular orbit at the periphery of a galaxy, and basically confirmed the general findings presented in [14]; note that our simulations have been made with about one million particles, while the earlier simulations had been car-

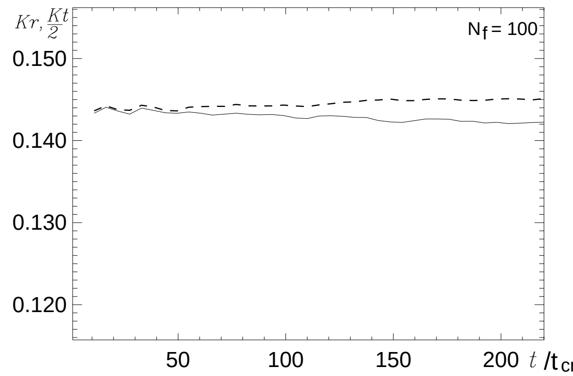

Fig. 4. The development of pressure anisotropy in a galaxy as a result of the interaction with a shell of $N = 100$ fragments dragged in toward the galaxy center by dynamical friction. The broken line represents the evolving value of $K_T/2$, where K_T is the total kinetic energy associated with the star motions in the tangential directions; the solid line represents the evolving value of K_r, the total kinetic energy associated with the star motions in the radial direction (from [6])

ried out with a few thousand particles. Then we have proceeded to address a quasi-spherical problem in which the satellite is fragmented into many smaller objects (several runs were made with either 20 or 100 fragments), distributed on a spherical shell. The quasi-spherical symmetry that characterizes this study has the important advantage of allowing for a smoother framework, basically free from other effects unrelated to dynamical friction, such as those associated with lack of equilibrium in the initial configuration. Furthermore, with respect to the earlier studies of the orbital decay of a single satellite, our attention here is mostly shifted to measuring the evolution of the underlying structure of the hosting galaxies. One effect observed, while the fragments are slowly dragged in toward the center, is a general change in the stellar density distribution with respect to the initial polytropic basic state. Another expected effect that we have been able to quantify, starting from an initially isotropic distribution of stellar orbits, is the slow growth of a tangentially biased pressure anisotropy (see Fig. 4). All these slow dynamical evolution effects appear to be genuinely associated with the process of dynamical friction exerted by the stars on the minority component of heavier objects. We are planning a survey of cases that should allow us to identify general properties of dynamical evolution in elliptical galaxies resulting from the interaction between the stars and a significant population of globular clusters or of the merging of a large number of small satellites.

4 General trends from thermodynamical arguments

In order to study possible general trends that may be anticipated for the evolution of elliptical galaxies, we refer to the general framework that has been successfully applied to the context of the evolution of globular clusters. Globular clusters appear to be well represented by King [34] models (see [24]). They are recognized to be non-homologous stellar systems, subject to dynamical evolution resulting from internal effects (such as weak collisionality and evaporation) and external perturbations (such as disk-shocking, when, in our Galaxy, their orbits happen to cross the disk). It has been noted that these mechanisms of dynamical evolution make a globular cluster evolve approximately along the King equilibrium sequence (see [47] and references therein).

For globular clusters, an important paradigm is provided by the *gravothermal catastrophe* [37], which offers interesting applications and physical interpretation (for a review, see [42]). Here we recall that, starting from the study of isothermal gas spheres [13], the gravothermal catastrophe is expected to occur also in stellar systems (see [1], [37]). The instability is interpreted as due to the curious property of self-gravitating systems of being characterized by a negative effective specific heat. Although for stellar systems a rigorous proof has been provided only for idealized models where an isothermal set of stars is confined by a spherical box, the paradigm is generally believed to be sufficiently robust to be applicable to real stellar systems, provided that they possess a sufficient level of internal collisionality. An independent element that strengthens the view that the paradigm is indeed robust has been added by an analysis that has shown, for an isothermal gas, that spherical symmetry is not a necessary ingredient [35].

Following some arguments initially put forward by Lynden-Bell (see [36], [37]), would there be a way to lay out a similar scenario for elliptical galaxies as partially relaxed stellar systems? If so, we would gather powerful "thermodynamical" arguments to determine general trends for evolution, beyond the specific paths produced by a given dynamical mechanism.

In our view, there are two aspects of the problem that require clarification. A first point is that we would like to start from a physically justified equilibrium sequence, much like King models for globular clusters, able to describe the general properties of elliptical galaxies. A second point is that, formally, the origin of the gravothermal catastrophe can be traced to the Poincaré stability of linear series of equilibria (see [31], [32]). For a proper mathematical derivation, one would thus like to start from a sequence of collisionless models derived rigorously from the Boltzmann entropy. In the absence of such a sequence, a derivation of the gravothermal catastrophe has been based on either an *unjustified ansatz* (see [33], [38]), that the global temperature of the system would be associated with the coefficient multiplying the energy in the distribution function, or the use of non-standard entropies [21] (but for unrealistic models).

In order to address the first point, we may refer to a sequence of models that have been found to be very promising for a realistic description of elliptical galaxies (the so-called f_∞ models; [9], see the review [11]). These models have been inspired by the characteristics of the products of collisionless collapse, as derived from N-body simulations [45]. In the simple spherical case, they are based on the distribution function $f_\infty = A(-E)^{3/2} \exp(-aE - cJ^2/2)$, with A, a, c positive constants, and define a one-parameter equilibrium sequence, which, much like

King models, can be parameterized in terms of the dimensionless central potential $\Psi = -a\Phi(0)$. For positive values of E the distribution function is taken to vanish. When Ψ increases beyond a certain value, around $\Psi = 7$, the models have a projected mass density profile that is well fitted by the $R^{1/4}$ law and indeed they turn out to be an excellent tool to fit the observations. From the point of view of statistical mechanics, they have been found [43] to be compatible with a derivation based on a partition of phase space in terms of the star energy and the star angular momentum square, under the assumption that detailed conservation of the star angular momentum is required at large values of angular momentum. This closely follows our understanding of the process of partial violent relaxation [36]. Unfortunately, the derivation is based on heuristic arguments and the distribution function does not follow from a straightforward exact mathematical extremization of the Boltzmann entropy; in particular, the orbit time that acts as a weight to the cells in phase space is replaced, for simplicity, by a factor $1/(-E)^{3/2}$, which is approximately correct only for weakly bound orbits. Therefore, attempts at using this equilibrium sequence to study the gravothermal catastrophe in the context of elliptical galaxies, while definitely appealing from the physical point of view (see also [8]), would remain less satisfactory from the formal point of view.

Now we have shown [12] that we can carry out a program that is satisfactory not only from the physical point of view (because it is based on an equilibrium sequence, also inspired by studies of collisionless collapse [45], that is able to match the properties of observed galaxies), but also from the mathematical point of view (because the distribution is derived rigorously from the Boltzmann entropy by requiring the conservation of a third global quantity Q, in addition to total energy and total mass). The program is made possible by the second option explored in [43] for the construction of models of partially relaxed stellar systems. This option leads to the so-called $f^{(\nu)}$ models. It was already noted [43] that the general physical properties of the $f^{(\nu)}$ models are close to those of the f_∞ models and, in particular, that for ν in the range $0.5-1$ their projected mass distribution, for concentrated models, follows the $R^{1/4}$ law.

Let f be the single-star distribution function, E the single-star specific energy, and J the magnitude of the single-star specific angular momentum. Consider the standard Boltzmann entropy:

$$S = -\int f \ln f d^3v d^3x \tag{2}$$

and extremize it under the constraint that the total mass

$$M = \int f d^3v d^3x, \tag{3}$$

the total energy

$$E_{tot} = \frac{1}{3}\int E f d^3v d^3x, \tag{4}$$

and a third global quantity

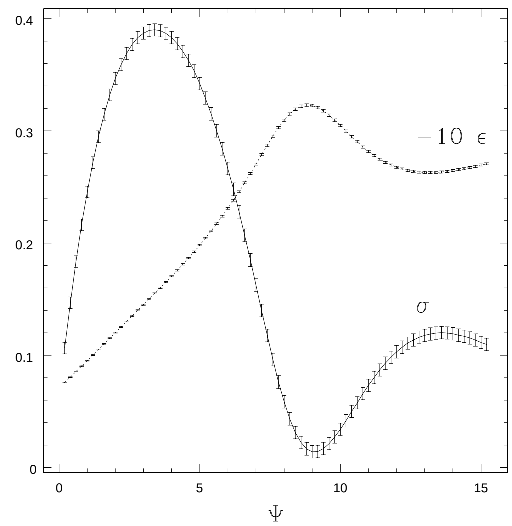

Fig. 5. Specific entropy and total energy along the equilibrium sequence of $f^{(\nu)}$ models with $\nu = 1$ (as a function of the concentration parameter Ψ, at constant M and Q, and thus expressed by means of the dimensionless functions $\sigma(\Psi)$ and $\epsilon(\Psi)$). Note that for $\Psi < 3.5$ the models are characterized by a negative global temperature, because the derivatives of S and E_{tot} have opposite signs. This figure is taken from [12]

$$Q = \int J^{\nu} |E|^{-3\nu/4} f d^3 v d^3 x \tag{5}$$

are assigned. Then the resulting distribution function is

$$f^{(\nu)} = A \exp\left(-aE - dJ^{\nu}|E|^{-3\nu/4}\right). \tag{6}$$

In the above expression, the quantities A, a, and d are positive constants. The parameter ν is a free (positive) parameter, which was argued [43] to be in the range $0.5 - 1.0$. In the following we refer to the case $\nu = 1$. Note that the three constants appearing in the distribution function define two scales and one dimensionless parameter, which we take to be $\gamma = ad^{2/\nu}/(4\pi GA)$.

Self-consistent models generated by such distribution function are computed from the Poisson equation, solved under the boundary conditions of regular potential at the center and of Keplerian potential at very large radii. For positive values of E the distribution function is taken to vanish. If we introduce the dimensionless central potential $\Psi = -a\Phi(0)$, the outer boundary condition defines a sort of eigenvalue problem that is solved by the relation $\gamma = \gamma(\Psi)$. The self-consistent models thus make a one-parameter equilibrium sequence.

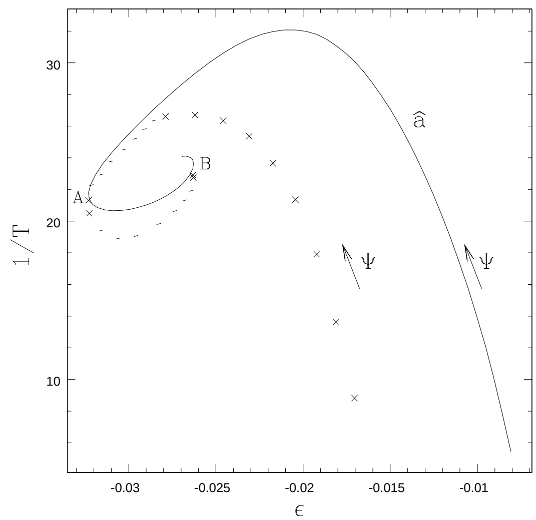

Fig. 6. The instability "spiral" of $f^{(\nu)}$ models with $\nu = 1$. The solid line refers to the results obtained from the *ansatz* that the coefficient a represents the inverse global temperature. Crosses represent the inverse global temperature from the definition $\partial S/\partial E_{tot}$; other symbols indicate estimated points for which the adopted numerical differentiation is less reliable. Point A marks the onset of the gravothermal catastrophe (from [12])

By careful numerical integration, one may then proceed to calculate the functions $S = S(M, Q, \Psi)$ and $E_{tot} = E_{tot}(M, Q, \Psi)$ on the equilibrium sequence (see Fig. 5) and from here the inverse *global temperature*

$$\zeta = \left(\frac{\partial S}{\partial E_{tot}} \right)_{M,Q} . \tag{7}$$

The onset of the gravothermal catastrophe is thus determined by inspection of the equilibrium sequence studied in the (E_{tot}, ζ) plane (following [31]).

In [12] we have implemented the above program and shown that for the $f^{(\nu)}$ models the gravothermal catastrophe is expected to set in at $\Psi \approx 9$. Surprisingly, around this value of the concentration, the projected mass distribution turns out to be very well fitted by the $R^{1/4}$ law (this general point had already been noted in [43], but outside the context of the gravothermal catastrophe). For values of Ψ close to and beyond 9, the general properties of the instability "spiral" in the (E_{tot}, ζ) plane, based on the proper thermodynamical definition of the global temperature, are the same as in the $(E_{tot}, \hat{a})$ plane, based on the *ansatz* that the temperature of the models is determined by the coefficient a (see Fig. 6).

One important point noted in [12] is a qualitative departure of the behavior of the instability "spiral" at low values of Ψ. For the original gas sphere and for the stellar dynamical analogue of a stellar system confined by a box with reflecting walls, the limit of low concentration was identified as that of a *non-gravitating ideal gas*, subject to Boyle's law. In our case, the analogy breaks down. In fact, the global temperature turns out to *change sign* at $\Psi \approx 3.5$ (see Fig. 5). Such a drastic event should be accompanied by some physical counterpart in the dynamical behavior of the system. Surprisingly, the value of $\Psi \approx 3.5$ coincides with that for the threshold of the radial-orbit instability [39] (for the context of f_∞ models, see [7] and [10]). In other words, by undertaking a thermodynamical description of the equilibrium sequence of models defined by the $f^{(\nu)}$ distribution function, we have found arguments that lead us naturally not only to the interpretation of the observed $R^{1/4}$ law, but also to one clue for the interpretation of the radial-orbit instability of collisionless stellar systems. Besides the properties just outlined, one important additional aspect that makes the $f^{(\nu)}$ models, at this point, more appealing than the f_∞ models is their anisotropy level. We had noted (e.g., see [11]) that the f_∞ models are actually too isotropic, when compared with the final products of simulations of collisionless collapse [45]. The present models turn out to be much more interesting even in this respect. We have indeed checked that their characteristic anisotropy profile, for values of Ψ close to the onset of the gravothermal catastrophe, is very similar to that observed in the numerical simulations (see Fig. 7).

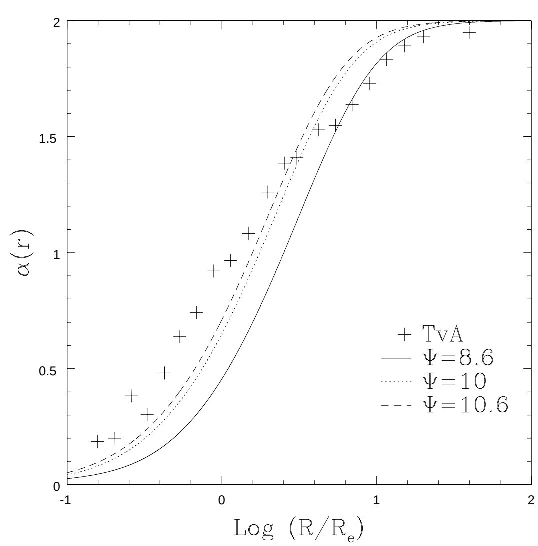

Fig. 7. Pressure anisotropy profiles $\alpha = 2 - (\langle v_\phi^2 \rangle + \langle v_\theta^2 \rangle)/\langle v_r^2 \rangle$ as a function of radius for selected $f^{(\nu)}$ models ($\nu = 1$) compared to the pressure anisotropy profile found [45] in numerical simulations of collisionless collapse. This figure has been prepared by M. Trenti

5 Conclusions

For a physically justified family of equilibrium models, representing the result of incomplete violent relaxation, and derived rigorously from the Boltzmann entropy, we have shown that, at high concentration values, the onset of the gravothermal catastrophe is found to occur at $\Psi \approx 9$, in the parameter domain where models are characterized by an $R^{1/4}$ projected density distribution. At low concentration values, the equilibrium sequence presents a drastic departure from the limit of the classical isothermal sphere, because models become associated with a negative global temperature. The transition point, $\Psi \approx 3.5$, turns out to coincide with the point of the sequence where the radial-orbit instability sets in. In the intermediate concentration regime, $3.5 < \Psi < 9$, the structural properties of the models change, much like those of models along the King equilibrium sequence, a family of models that is known to capture the non-homologous properties of globular clusters. It is our hope that, in this domain of intermediate concentration values, the $f^{(\nu)}$ models may be used to describe the characteristics of weak homology of elliptical galaxies.

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